Doklady Physics, Vol. 50, No. 10, 2005, pp. 490–493. Translated from Doklady Akademii Nauk, Vol. 404, No. 4, 2005, pp. 477–480.

PHYSICS

Stabilization of Turbulent Dynamics in Excitable Media by an External Point Action

A. Yu. Loskutov, R. V. Cheremin, and S. A. Vysotskiĭ

Presented by Academician A.R. Khokhlov May 13, 2005

Received May 16, 2005

In this paper, the behavior of an excitable medium in the state of developed spatio-temporal chaos is analyzed. We show that a weak point action on the medium results in the suppression of all spiral waves and stabilization of the system dynamics. The analysis performed is based on the identification of the number of spiral waves in the medium.

The stabilization of the turbulent dynamics of active media, which is based on a weak periodic point action, is a rather important direction of research, finding its application in cardiology. At present, in the theory of excitable systems, a hypothesis dominates according to which the appearance of a fatal cardiac arrhythmias is associated with the generation in the heart tissue of a great number of autowave sources, namely, spiral waves and vortex structures (i.e., spatio-temporal chaos) (see, e.g., [1, 2] and references therein).

Modern methods for the stabilization of such regimes based on single electric pulses (including those based on implanted defibrillators) are rather arduous and not always successful. However, recent studies open novel potentialities in this field of medical science. There is no necessity in high-amplitude pulsed action, and, in many cases, the action can be weakened [3]. Moreover, in a number of excitable media, the turbulent regime can be stabilized by a rather weak periodic parametric action [4, 5] or by a force action applied at a certain domain of a medium [6–8].

In this study, we exploit the simple theoretical model of the FitzHugh–Nagumo type [9, 10] for an excitable medium. We show that the spatio-temporal chaos arising as a result of the decay of spiral waves can be suppressed by means of a point action having a rather low amplitude. In addition, the problem of seeking frequencies that provide for the efficient suppression of all spiral waves is solved. Upon this stabilization, the medium remains in the spatially homogeneous state.

The FitzHugh–Nagumo model describes a two-component system of the activator–inhibitor type:

$$\frac{\partial U}{\partial t} = \Delta U - U(U-\alpha)(U-1) - V,$$
$$\frac{\partial V}{\partial t} = \beta U - \gamma V. \tag{1}$$

As applied to the heart-muscle dynamics, variable U corresponds to the action potential for muscular cells.

Although this model well describes (at the qualitative level) the excitation propagation in the muscular tissue and demonstrates basic types of structures arising in excitable media of the activator–inhibitor type, it is unsuitable for quantitative description. This is associated with the fact that this model does not allow for certain important properties of the heart tissue, such as the dependence of the refractoriness period on both the amplitude and duration of the excitation phase.

In order to obtain a more adequate description, the set of Eqs. (1) is usually represented in the form

$$\frac{\partial U}{\partial t} = \Delta U - f(U) - V,$$

$$\frac{\partial V}{\partial t} = g(U, V)(kU - V).$$

In this case, the form of the functions f and g is chosen with a goal to providing for the consistency of the action-potential profiles to be obtained with experimental data.

Recently, the model developed in [11] has been widely employed, it having been proposed there that

Moscow State University,
Vorob'evy gory, Moscow, 119992 Russia
e-mail: loskutov@moldyn.phys.msu.ru

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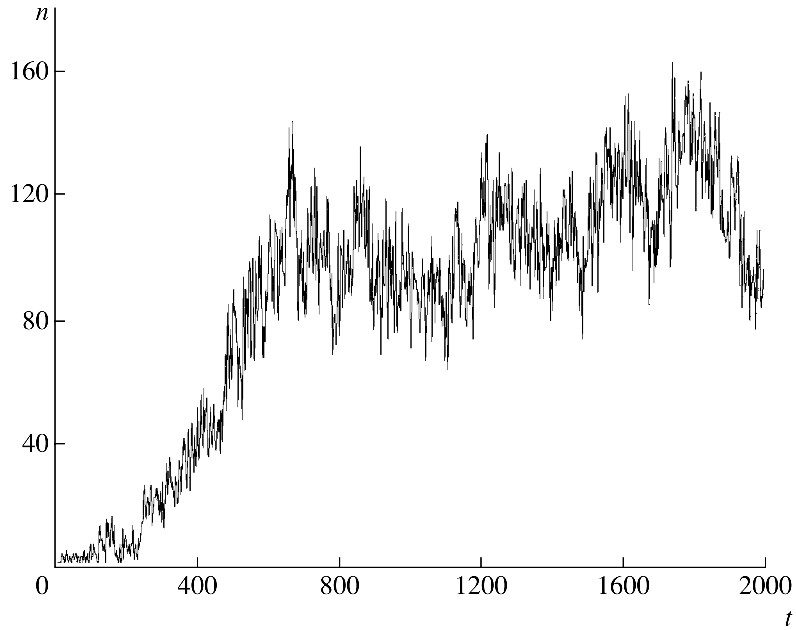

Fig. 1. Number of spiral-wave cores as a function of time in the case of the decay of a single spiral wave and generation of chaos ($G_1 = 1/50$, $G_3 = 0.3$).

the following piece-wise linear functions be used for the functions f and g:

$$f(U) = \begin{cases} C_1 U, & U < U_1, \\ -C_2 U + a, & U \in (U_1 U_2), \\ C_3(U-1), & U > U_2, \end{cases}$$

$$g(U,V) = \begin{cases} G_1, & U < U_1, \\ G_2, & U_1 > U_2, \\ G_3, & U < U_1,\ V < V_1. \end{cases} \quad (2)$$

One of the advantages of this description is the presence of two independent relaxation parameters. One of them (G_3) determines the relaxation period for small values of U and V. The other parameter (G_1) determines the absolute value of the relaxation parameter for large values of V and intermediate values of U, which corresponds to the leading and trailing wave fronts.

To ensure a correspondence with actual media (e.g., heart tissue), the following values of the parameters related to the set of Eqs. (2) are usually chosen: $C_1 = 20$, $C_2 = 3$, $C_3 = 15$, $U_1 = 0.0026$, $U_2 = 0.837$, $V_1 = 1.8$, $a = 0.06$, and $k = 3$. In this case, G_1 ranges from $\frac{1}{75}$ to $\frac{1}{33}$, $G_2 = 1$, and $0.1 \le G_3 \le 2$.

We have analyzed the dynamics of this system in a rectangular plane domain of the size of 350×350 nodes. In order to exclude edge effects at the boundaries, we set the periodic conditions; i.e., the domain under study had the torus topology.

In the chosen range of parameter values, the autowave solutions of the spiral-wave type are unstable. As time elapses, they decay into smaller waves and, as a result, the regime of spatio-temporal chaos is developed in the system. The spiral waves are the basic types of the autowave solutions in the given system. This opens the possibility of using their number as a criterion of the complexity of the regime existing in the system [12–15]. The calculation algorithm is based on the fact that the core of a spiral wave (as an arbitrary singularity point of the wave-front) is a singularity for the phase field:

$$\varphi(x, y, t) = \arctan 2\,(U(x, y, t) - U^*, V(x, y, t) - V^*).$$

In this case, the quantity $n = \frac{1}{2\pi}\oint \nabla\varphi dl$, called the topological charge, differs from zero only when the integration contour envelopes the singularity. This is the case when n is an integer, and its sign determines the chirality of the spiral wave. The time dependence for the calculated number of spiral-wave cores is plotted in Fig. 1. The plot corresponds to the appearance of the chaotic regime arising from a decaying single spiral wave.

This chaotic regime is further used as the initial state in the analysis of a system with a point periodic action of the rectilinear-step shape:

$$I_{\mp}(t) = A(2\theta(t - T\tau) - 1).$$

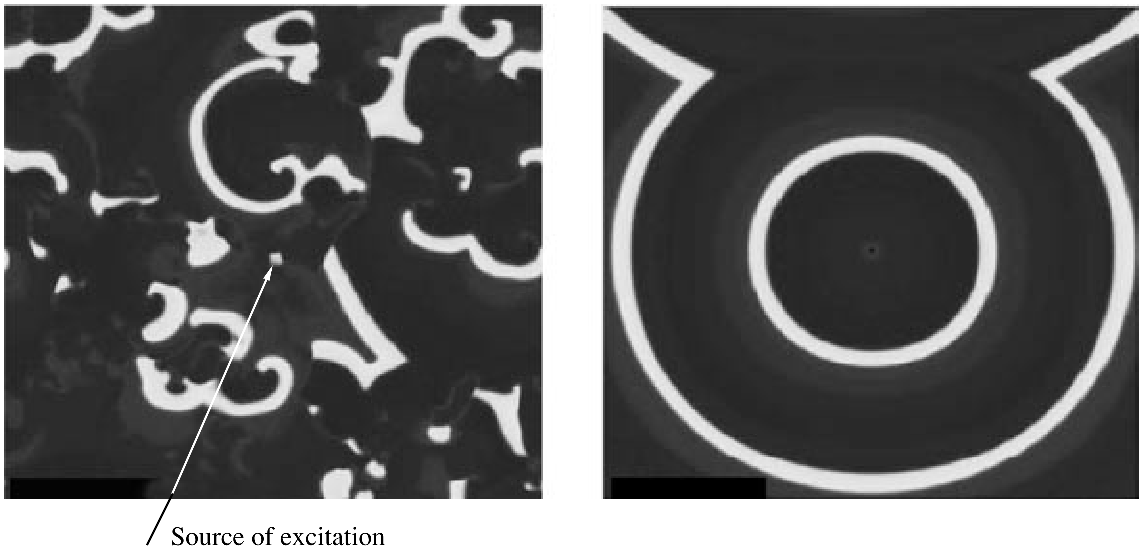

Fig. 2. Results of the action on a system with developed spatio-temporal chaos (G_1 = 1/50, G_3 = 0.3, A = 6).

Here, A = 6 is the amplitude, θ is the Heaviside step function, and τ varies within the range from 0.1 to 0.9. The action was applied to the domain enveloping 2 × 2 nodes. As applied to the heart tissue, this action is weaker than in the case of an implanted defibrillator by a factor of 1000.

Insofar as arbitrarily (in the dark) seeking suppression frequencies is extremely inefficient, we employed a method that made it possible to preliminarily localize the range of frequencies providing the suppression. The concept of this method is based on a well-known property of excitable media: in the case of competing wave sources, only the source with the highest frequency of the generated waves survives. Thus, the most favorable frequencies of the external action (from the standpoint of suppressing spiral waves) are those for which the frequency of excited circular waves in the medium is close to the maximum possible frequency for the given parameters of the medium.

In order to determine these frequencies, we generated circular waves in a small volume of the medium and found the dependence of the frequency ν of the waves being obtained on the frequency ω of the point source. The frequency intervals in the vicinity of the maxima of this dependence served as candidates for a more detailed study. It is worth noting that the simulation in a small volume of the medium over the course of several tens of periods is sufficient for constructing the function $\nu(\omega)$. However, to verify the presence or absence of the suppression effect at the given frequency, it is necessary to consider large volumes of the medium over the course of several thousands of periods, since otherwise the turbulent regime is not developed.

The set of Eqs. (2) was investigated for values of the parameters G_1 = {1/75, 1/50, 1.33} and G_3 = {0.1, 0.3, 0.5, 1.0, 1.5, 2.0}, i.e., on the calculation mesh with 18 nodes. For all of them, the effect of suppression of the spatio-temporal chaos was observed (Fig. 2) at frequencies in the vicinity of the maximum for the function $\nu(\omega)$ (Fig. 3). The number of cores for spiral waves as a function of time in the system with a point action is shown in Fig. 4.

The numerical analysis performed has demonstrated that the stabilization of the dynamics is also possible if two or more excitation sources are introduced into the medium. However, in this case, the suppression efficiency noticeably depends on the distance between the sources. When they are spaced for a sufficient distance, the effective action turns out to be stronger than in the

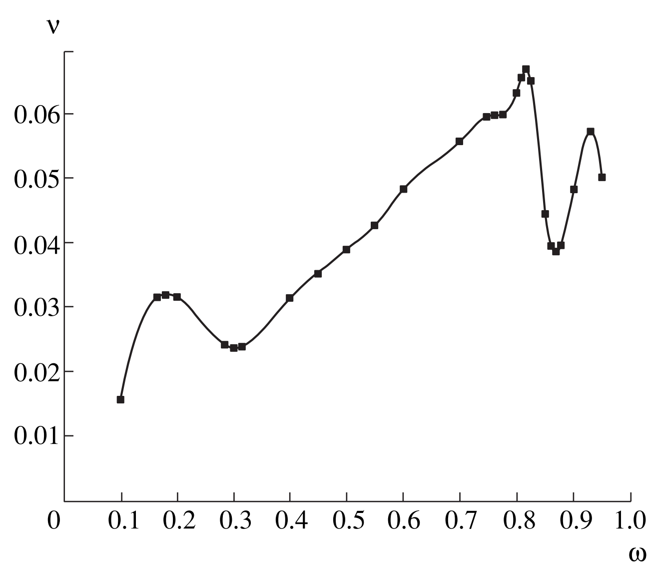

Fig. 3. Dependence $\nu(\omega)$ (G_1 = 1/50, G_3 = 0.3).

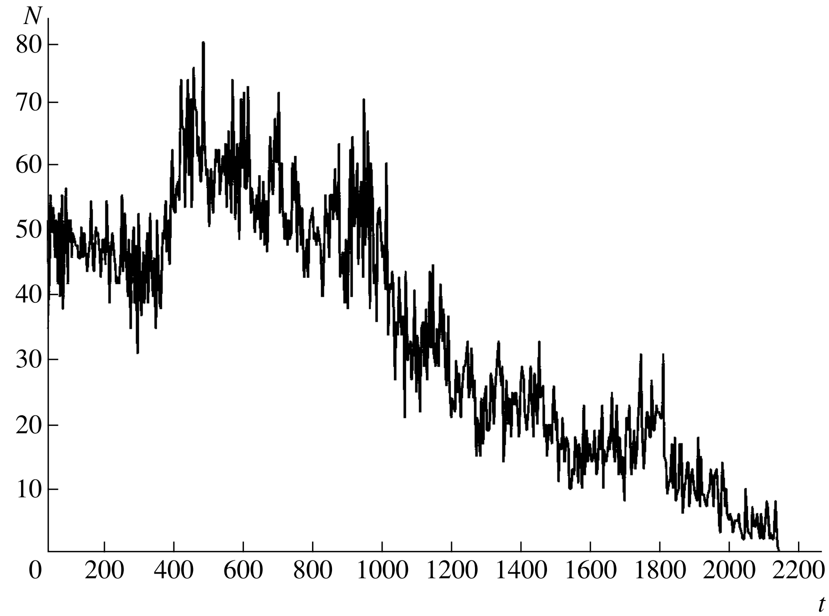

Fig. 4. Number of spiral waves as a function of time in the case of the point action ($G_1 = 1/50$, $G_3 = 0.3$, $A = 6$).

case of a single source, so that the suppression of spiral waves occurs more rapidly by a factor of 3–5. If the sources are placed closely to each other, they begin to compete, thereby terminating the generation of circular waves.

An additional series of numerical experiments was carried out with an extended source in the form of a thin filament. It was found that the suppression efficiency rapidly dropped as the filament length was increased. For example, filaments of length $10l$ and longer, where l is the wave-front width, yielded the inverse effect; namely, the number of spiral-wave cores increased.

Thus, the most efficient method for the stabilization of the turbulent dynamics of excitable media is that of suppressing spiral waves by a weak point action in the form of a single or several sufficiently spaced small-size sources. In the future, we hope to find conditions that will allow us to reduce even more the amplitudes of the negative half-wave. This will result in a decrease in the total power of the action due to choosing a special pulse shape, and, hence, will allow us to suppress the spatio-temporal chaos by means of purposefully chosen low-intensity pulses. It would appear to be impossible to attain this effect with ordinary sinusoidal-shape pulses.

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Translated by G. Merzon