Doklady Physical Chemistry, Vol. 396, Part 1, 2004, pp. 105–107. Translated from Doklady Akademii Nauk, Vol. 396, No. 1, 2004, pp. 68–70.

PHYSICAL CHEMISTRY

Stabilization of Diffusion-Induced Chaotic Processes

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Presented by Academician A.R. Khokhlov December 30, 2003

Received January 14, 2004

In this paper, it is shown that diffusion-induced chaos in a distributed system can be suppressed by feedback-free parametric action.

The possibility of stabilizing chaotic oscillations was described for the first time by, most likely, Alekseev and Loskutov [1, 2], who considered a class of ordinary differential equations with nonpolynomial right sides. Later, a feedback-free method for suppressing chaos in systems of a certain class was analytically substantiated by Loskutov and Shishmarev [3, 4] (see also Loskutov [5] and references therein). This subject became popular after publication of a paper by Ott *et al.* [6], who showed that quite weak feedback parametric perturbations can suppress chaos and stabilize virtually any saddle limit cycle embedded in a chaotic attractor. Most studies on this subject deal with stabilization of the dynamics of lumped systems. At the same time, studies of the possibility of suppressing *spatiotemporal chaos* are few, although approaches to very important applications can be found in this way (see [7–9] and references therein).

In this paper, to reveal parametric stabilization of the chaotic behavior, we studied the distributed reaction–diffusion system described by the set of equations [10–12]

$$\partial_t u = D\partial_{zz}u + \frac{1}{\varepsilon}\left(A(z)u - u^2 + \frac{q(z)B(z)(pA(z)v - uv)}{pA(z) + u}\right),$$
$$\partial_t v = D\partial_{zz}v + A(z)u - B(z)v. \qquad (1)$$

This set is known as the diffusion Oregonator and describes one of the types of the Belousov–Zhabotinsky reaction in the Couette reactor. Here, A is the concentration of BrO_3^- (the initial reactant), B is the concentration of organic species (the initial reactants), u is the $HBrO_2$ concentration, v is the $[Ce^{4+}]$ concentration, q is the number of Br^- ions per Ce^{4+} ion, ε and p are stoichiometric factors, and the spatial coordinate z is normalized to unity.

Model (1) has only two independent variables and, therefore, cannot have chaotic properties. However, diffusion leads to instability and, as a consequence, to the development of a chaotic dynamics. Such a phenomenon is called diffusion-induced chaos.

For the Couette reactor, one can assume [13] that the spatial profiles of the concentrations A and B are determined only by diffusion and, consequently, the parameters vary linearly with the axial coordinate z. The q value depends on the ratio between the concentrations of the organic and bromine-containing species and, therefore, also varies linearly with the spatial coordinate:

$$A(z) = A(0)\left(1 - \frac{5}{6}z\right), \quad B(z) = \frac{B(1)}{6}(1 + 5z),$$
$$q(z) = 1.5 - 0.6z. \qquad (2)$$

The stoichiometric factors ε and p are constant since they depend only on reaction rates: $\varepsilon = 2.2 \times 10^{-2}$ and $p = 3.5 \times 10^{-3}$ [13]. The effective diffusion coefficient D is assumed to be the same for the two variables u and v: $D = 3.75 \times 10^{-4}$. The conditions at the reactor boundary have the form

$$\left.\frac{\partial u}{\partial z}\right|_{z=0} = \left.\frac{\partial u}{\partial z}\right|_{z=1} = \left.\frac{\partial v}{\partial z}\right|_{z=0} = \left.\frac{\partial v}{\partial z}\right|_{z=1} = 0. \qquad (3)$$

Thus, the Oregonator in the Couette reactor can be described by boundary value problem (1) under boundary conditions (3).

Analysis of set (1)–(3) showed that, depending on the values of the control parameters, its solution $v(z, t)$, $u(z, t)$ can be rather complex (quasi-periodic and chaotic) only within the spatial range $0 < z < 0.5$. At the same time, within the range $0.5 < z < 1$, at any admissible parameter values under the given diffusion, the system is either in a steady state or exhibits only periodic dynamics. This is explained by the fact that self-oscillations excited at $z = 0$ rapidly decay as they propagate along the reactor.

As a bifurcation parameter, one can choose the quantity $A(0)$ (see expressions (2)) at fixed $B(1)$. The behavior of the system at different $A(0)$ is illustrated in

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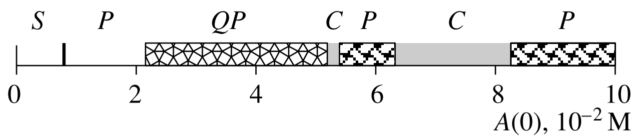

Fig. 1. Bifurcation diagram of set (1)–(3) at $B(1) = 0.85$: *S*, stationary point; *P*, periodic behavior (limit cycle); *QP*, quasi-periodic behavior; *C*, chaotic behavior.

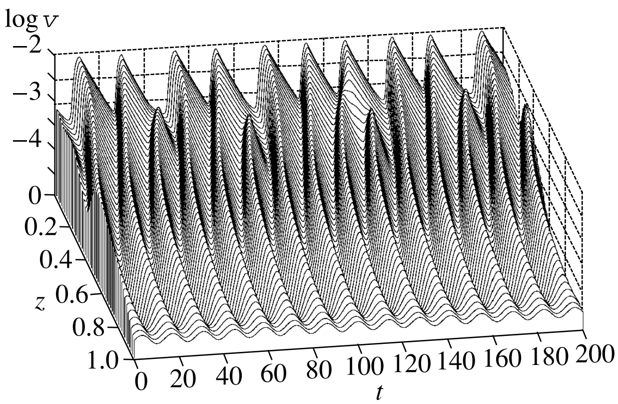

Fig. 2. Dynamic pattern of oscillations in set (1)–(3) at $A(0) = 7.0 \times 10^{-2}$ (chaos).

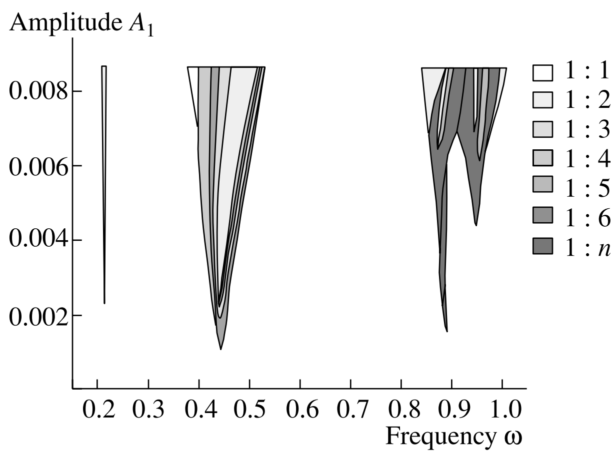

Fig. 3. Regions of the stabilized behavior of set (1)–(4) in the parametric space (A_1, ω).

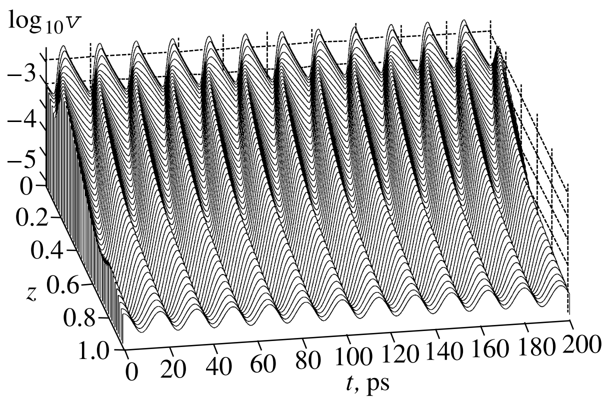

Fig. 4. Spatiotemporal dynamics of set (1)–(3) under external action (4) at $A_0 = 7.2 \times 10^{-2}$, $A_1 = 0.0081$, and $\omega = 0.3855$ (stabilized behavior).

Fig. 1. At $A(0) < 0.8 \times 10^{-2}$, the system tends to a steady state. At $A(0) \in (0.8 \times 10^{-2}; 2.3 \times 10^{-2})$ in the range $0 < z < 0.5$, the solution has the form of periodic oscillations with time. At $A(0) > 2.3 \times 10^{-2}$, oscillations propagate from the reactor inlet along the reactor so that, at z close to 1, they are also periodic. With an increase in $A(0)$, the mode at small z is complicated (Fig. 2) and becomes sharper and more disordered.

The modes of the behavior of the system were identified by well-known methods, such as the methods of Lyapunov indices, the power spectrum, and Poincaré mapping. By these methods, two regions of the chaotic behavior were determined: $A_c = (5.2 \times 10^{-2}; 5.3 \times 10^{-2})$ and $A_c = (6.5 \times 10^{-2}; 8.3 \times 10^{-2})$ (Fig. 1).

The possibility of stabilizing chaotic oscillations was studied by one of the simplest feedback-free parametric action methods, namely, harmonic perturbation of the parameter $A(0)$ in the chaos region:

$$A(0) = A_0 + A_1 \sin(\omega t), \quad A_0 = \frac{A_a + A_b}{2},$$

$$A_1 < \frac{A_b - A_a}{2}, \tag{4}$$

where A_1 and ω are the perturbation amplitude and frequency, respectively, and A_a and A_b are the values of the parameter $A(0)$ at the boundaries of the A_c region where there is a chaotic mode. It is easy to see that, when changing so at any A_1, the external perturbation remains within the chaos region for set (1)–(3).

Thorough numerical analysis revealed regions where the chaotic mode is suppressed and the mode of behavior of the system becomes regular. These regions have the shape of narrow wedges with a quite complex internal structure. This structure is formed by nested peaks of synchronization of different ratios (Fig. 3). At some values of the perturbation amplitude and frequency, the system and the external perturbation are fully synchronized. In Fig. 3, this region is not shaded. In this case, the phase portrait is a closed curve and the oscillation pattern for a fixed spatial curve $v(z_{\text{fix}}, t)$ is periodic. Regions corresponding to cycles with periods of 2, 4, etc., are shaded to different degrees. Parameter regions corresponding to the complex periodic behavior are colored black. The spatiotemporal pattern of oscillations at $A_0 < 7.2 \times 10^{-2}$, $\omega = 0.3855$, and $A_1 = 0.0081$ (Fig. 3, the unshaded region) is presented in Fig. 4.

Generalization of these results can be extremely important to applications where it is necessary to *gently* remove a system from a state of spatiotemporal chaos, e.g., in cardiology, where the development of chaos is fatal to the organism. Modern methods for stabilizing highly chaotic heart rhythms (fibrillation) are very harsh: a brief electric pulse of high voltage and current. However, preliminary studies show that weak periodic action is likely to be able to achieve stabilization and restore the rhythm. This is also possible in systems of quite different origin [14, 15].

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