Journal of Experimental and Theoretical Physics, Vol. 98, No. 5, 2004, pp. 1045–1053.
Translated from Zhurnal Éksperimental'noĭ i Teoreticheskoĭ Fiziki, Vol. 125, No. 5, 2004, pp. 1194–1203.

NONLINEAR PHYSICS

Suppression of Chaos in the Vicinity of a Separatrix

A. Yu. Loskutov and A. R. Dzhanoev
Moscow State University, Moscow, 119899 Russia
e-mail: loskutov@moldyn.msu.su
Received April 30, 2003

Abstract—The standard Melnikov method for analyzing the onset of chaos in the vicinity of a separatrix is used to explore the possibility of suppression of chaos of dynamical systems of a certain class. Analytical expressions are obtained for external perturbations that eliminate chaotic behavior. These results are supplemented with a numerical analysis of the Duffing–Holmes-oscillator and pendulum equations. © *2004 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

Intensive theoretical and experimental studies of chaotic dynamical systems revealed their unexpected and remarkable property: they are highly susceptible and extremely sensitive to perturbations. This discovery served as a starting point for finding a means to control the behavior of chaotic systems, i.e., to change from chaotic regimes to required regular oscillatory regimes by means of relatively weak perturbations.

Suppression of unstable or chaotic behavior of dynamical systems is generally achieved via stimulated excitation of stable (usually periodic) oscillations by means of multiplicative or additive perturbations. In other words, an external perturbation is required to change from a chaotic state of a system to a regular regime. The statement of the problem is outwardly simple, but its solution is very difficult to find for particular dynamical systems. Moreover, even though the problem has been analyzed in numerous studies, a systematic and rigorous theory of suppression of chaotic behavior has been developed only for some common families of dynamical systems (see [1, 2] and references cited therein).

Chaotic behavior can be suppressed by two different methods. In one of these, the state of a system is changed from chaotic to regular by perturbation without feedback. In other words, this method does not make use of the current values of dynamic variables. In the other method, the perturbation is adjusted in accordance with the required values of dynamic variables; i.e., feedback is an integral component of the dynamical system. By convention, the former method is called open-loop suppression (or control) of chaotic dynamics. The latter method is called feedback control of chaotic systems. Both methods can be implemented either parametrically or by direct forcing.

To the best of our knowledge, the first analyses of suppression of chaotic dynamics of certain systems were presented in [3, 4]. However, extensive research along these lines was initiated by [5, 6], where it was shown that relatively weak parametric perturbations can be used to regularize a particular saddle orbit embedded in a chaotic attractor. These and other results stimulated studies of suppression of chaotic dynamics and evoked great interest in control of unstable systems. A vast number of numerical and experimental studies were focused on the possibility of suppression of chaos and implementation of periodic or other required dynamics in various systems and maps (see [1, 2, 7–10] and references therein).

The standard Melnikov method is an effective tool used in analytical treatments of the problem of chaos suppression [11]. It is based on comparison of the first-order terms in the series expansions of the solution in terms of a perturbation parameter on stable and unstable separatrices. In particular, the Melnikov method was applied to explore the possibility of elimination of chaotic dynamics of the Duffing–Holmes oscillator [12–16] (see also [17]). It was shown that a small parametric perturbation of the system's chaotic dynamics suppresses chaos. Furthermore, the Melnikov method was used in [18] to examine the effects of parametric perturbations in a model of the Josephson junction.

In this paper, the Melnikov method [11, 19] is applied to find analytical expressions for parametric perturbations that suppress chaotic and/or unstable behavior of dissipative dynamical systems. The Duffing–Holmes oscillator and pendulum are considered as examples.

2. THE MELNIKOV METHOD

In this section, we briefly describe the Melnikov analytical method for identifying homoclinic or heteroclinic chaos, relying on the original paper [11] (see also [19–21]).

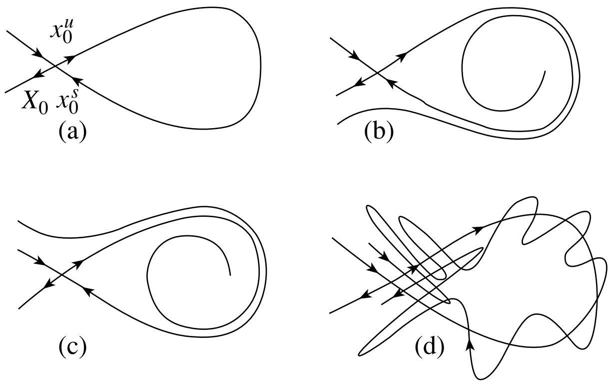

Fig. 1. Split separatrix loops.

Consider a simple autonomous system with a single hyperbolic point X_0 subject to a periodic perturbation:

$$\dot{x} = f_0(x) + \varepsilon f_1(x, t), \quad (1)$$

where $x = (x_1, x_2)$ and f_1 is a periodic function with period T. Suppose that the unperturbed system (with $\varepsilon = 0$) has a single separatrix $x_0(t)$ (see Fig. 1a):

$$\lim_{t \to \pm\infty} x_0(t) = X_0.$$

The separatrix is split by the perturbation, i.e., has distinct incoming and outgoing branches. Three possibilities arise as a result: the separatrices either do not intersect (in which case one may enclose the other, see Figs. 1b and 1c) or intersect at an infinite number of homoclinic points. Chaotic dynamics are observed only in the latter case (see Fig. 1d).

To find an intersection condition, one must use a perturbation method to calculate the distance $D(t, t_0)$ between the separatrices at an instant t_0. If the outgoing separatrix encloses the incoming one, then $D(t, t_0) < 0$. If the incoming separatrix encloses the outgoing one, then $D(t, t_0) > 0$. Only if there exists t_0 such that the separatrices intersect, then the sign of $D(t, t_0)$ alternates.

In the method substantiated in [11], the distance $D(t, t_0)$ between the branches of a split separatrix is determined by performing integration along unperturbed trajectories. The method is based on comparison of the first-order terms in the series expansions of the solution in terms of the perturbation parameter ε on stable and unstable separatrices.

To calculate $D(t, t_0)$, it is sufficient to find the solutions on the stable and unstable manifolds, x^s and x^u. When $\varepsilon = 1$, these solutions differ by the vector

$$r(t, t_0) = x^s(t, t_0) - x^u(t, t_0) = x_1^s(t, t_0) - x_1^u(t, t_0).$$

The Melnikov distance is the projection of r on the direction normal to the unperturbed separatrix x_0 at an instant t.

Omitting intermediate calculations, we write out an expression for D:

$$D(t, t_0) = -\int_{-\infty}^{\infty} f_0 \wedge f_1 dt. \quad (2)$$

This function determines conditions for chaotic behavior of the original system. In the domain where the sign of $D(t, t_0)$ alternates, the separatrices intersect and the system exhibits chaotic dynamics.

3. ELIMINATION OF CHAOTIC DYNAMICS IN THE VICINITY OF A SEPARATRIX

We use the mathematical procedure described above to explore the possibility of suppressing chaotic dynamics for systems with separatrix loops described by Eq. (1), where

$$f_0(x) = (f_{01}(x), f_{02}(x)),$$

$$f_1(x) = (f_{11}(x, t), f_{21}(x, t)). \quad (3)$$

For such a system, the Melnikov function $D(t, t_0)$ can be written as

$$D(t, t_0) = -\int_{-\infty}^{\infty} f_0 \wedge f_1 dt \equiv I[g(x, t)].$$

Suppose that the sign of $D(t, t_0)$ alternates, i.e., the separatrices intersect (see Fig. 1d). We seek a perturbation $f^*(\omega, t)$ that eliminates the intersection of the separatrices:[1]

$$\dot{x} = f_0(x) + \varepsilon[f_1(x, t) + f^*(\omega, t)], \quad (4)$$

where

$$f^*(\omega, t) = (f_1^*(\omega, t), f_2^*(\omega, t)).$$

We denote by $[s_1, s_2]$ the interval where the sign of $D(t, t_0)$ alternates. Two cases can arise when the system is perturbed by $f^*(\omega, t)$:

$$D^*(t, t_0) > s_2 \quad (5)$$

or

$$D^*(t, t_0) < s_1, \quad (6)$$

where $D^*(t, t_0)$ is the Melnikov distance for system (4). Suppose that (5) is satisfied. (A similar analysis can be

[1] We tentatively call f^* a regularizing perturbation.

performed when inequality (6) holds.) Then,

$$I[g(x,t)] + I[g^*(\omega, x, t)] > s_2. \quad (7)$$

where

$$I[g^*(\omega, x, t)] = -\int_{-\infty}^{+\infty} f_0 \wedge f^* dt.$$

By virtue of (7), there exists χ such that

$$I[g(x,t)] + I[g^*(\omega, x, t)] = s_2 + \chi = \text{const},$$
$$\chi, s_2 \in \mathbb{R}^+.$$

Hence,

$$I[g^*(\omega, x, t)] = \text{const} - I[g(x,t)]. \quad (8)$$

On the other hand,

$$I[g^*(\omega, x, t)] = -\int_{-\infty}^{\infty} f_0 \wedge f^* dt. \quad (9)$$

Suppose that the function $f^*(\omega, t)$ is absolutely integrable over an infinite interval and Fourier transformable. We define $f^*(\omega, t)$ as

$$f^*(\omega, t) = \text{Re}\{\hat{A}(t)e^{-i\omega t}\}$$

with $\hat{A}(t) = (A(t), A(t))$, i.e., assume that the regularizing perturbations applied to both components of Eq. (4) are identical. Therefore,

$$-\int_{-\infty}^{\infty} f_0 \wedge \{\hat{A}(t)e^{-i\omega t}\} dt = \text{const} - I[g(x,t)].$$

The inverse Fourier transform yields

$$f_0 \wedge \hat{A}(t) = \int_{-\infty}^{\infty} (I[g(x,t)] - \text{const}) e^{i\omega t} d\omega.$$

Hence,

$$A(t) = \frac{1}{f_{01}(x) - f_{02}(x)} \times \int_{-\infty}^{\infty} (I[g(x,t)] - \text{const}) e^{i\omega t} d\omega.$$

The quantity $A(t)$ can be interpreted as the amplitude of a regularizing perturbation.

Thus, dynamics of systems that can be represented as (1), (3) are regularized by the perturbation

$$f^*(\omega, t) = \text{Re}\left[\frac{e^{-i\omega t}}{f_{01}(x) - f_{02}(x)} \int_{-\infty}^{\infty} (I[g(x,t)] - \text{const}) e^{i\omega t} d\omega\right].$$

Next, we explore the possibility of suppressing chaotic dynamics for systems governed by equations of the form

$$\begin{aligned} \dot{x} &= P(x,y), \\ \dot{y} &= Q(x,y) + \varepsilon[f(\omega,t) + \alpha F(x,y)], \end{aligned} \quad (10)$$

where $f(\omega, t)$ is a periodic perturbation; $P(x, y)$, $Q(x, y)$, and $F(x, y)$ are smooth functions; and α is a damping parameter.

We consider the most common case when a single hyperbolic point is located at the origin ($x = y = 0$) and $P(x, y) = y$. Let $x_0(t)$ be the solution on the separatrix. For perturbed system (10), the Melnikov distance can be represented as

$$D(t, t_0) = -\int_{-\infty}^{\infty} y_0(t - t_0) \times [f(\omega,t) + \alpha F(x_0, y_0)] dt \equiv I[g(\omega, \alpha)], \quad (11)$$

where $y_0(t) = \dot{x}_0(t)$. As in the case of Eq. (1), assume that the sign of the Melnikov distance for system (10) alternates, i.e., the separatrices intersect. We seek a perturbation $f^*(\omega, t)$ that eliminates chaotic dynamics:

$$\begin{aligned} \dot{x} &= y, \\ \dot{y} &= Q(x,y) + \varepsilon[f(\omega,t) + \alpha F(x,y) + f^*(\omega,t)]. \end{aligned} \quad (12)$$

Since system (10) is parameterized by α, chaos must be suppressed for each particular value of the parameter. Accordingly, we can write $I[g(\omega)]$ instead of $I[g(\omega, a)]$.

For system (12),

$$f_{01} = y, \quad f_{02} = Q(x,y), \quad \hat{A}(t) = (0, A(t)).$$

Therefore,

$$A(t) = \frac{1}{y_0(t - t_0)} \int_{-\infty}^{\infty} (I[g(\omega)] - \text{const}) e^{i\omega t} d\omega.$$

Thus, a regularizing perturbation for system (12) can be represented as

$$f^*(\omega, t) = \text{Re}\left[\frac{e^{-i\omega t}}{y_0(t - t_0)} \int_{-\infty}^{\infty} (I[g(\omega)] - \text{const}) e^{i\omega t} d\omega\right].$$

Now, let us find a regularizing perturbation in the case when the Melnikov function $D(t, t_0)$ admits an additive shift from its critical value.

Again, we analyze the case when (5) is satisfied. Suppose that α_c corresponds to the critical value of the Melnikov function,

$$I_c = I[g(\omega, \alpha|_{\alpha = \alpha_c})].$$

Then, a subcritical Melnikov distance can be expressed as

$$I_{out} = I_c - \alpha,$$

where $a \in \mathbb{R}^+$ is a constant. Assuming that the system perturbed by $f^*(\omega, t)$ exhibits regular behavior, we have

$$I' + I_{out} + I[g^*(\omega)] > s_2. \quad (13)$$

where

$$I[g^*(\omega)] = -\int_{-\infty}^{+\infty} y_0(t - t_0) f^*(\omega, t) dt.$$

On the other hand, it is obvious that we can take any I' a fortiori greater than I_c:

$$I' = I_c + a > s_2. \quad (14)$$

Now, equating the left-hand sides of (13) and (14), we obtain $I[g^*(\omega)] = 2a$. Substituting

$$f^*(\omega, t) = \mathrm{Re}\{A(t)e^{i\omega t}\},$$

into the expression for $I[g^*(\omega)]$, we find

$$-\int_{-\infty}^{\infty} e^{i\omega t} A(t) y_0(t - t_0) dt = 2a.$$

The inverse Fourier transform yields

$$A(t) y_0(t - t_0) = -2a \int_{-\infty}^{\infty} e^{-i\omega t} d\omega.$$

Hence,

$$A(t) = -\frac{2a}{y_0(t)} \int_{-\infty}^{\infty} e^{-i\omega t} d\omega = -\frac{4\pi a \delta(t)}{y_0(t - t_0)}.$$

Thus, dynamics of systems that admit additive shift from the critical value of the Melnikov function $D(t, t_0)$ are regularized by the perturbation

$$f^*(\omega, t) = -\frac{4\pi a \delta(t)}{y_0(t - t_0)} \cos(\omega t), \quad (15)$$

where $\delta(t)$ is the Dirac delta function.

In the general case, if $f_0 = (f_{01}(x), f_{02}(x))$, then we obviously obtain

$$f^*(\omega, t) = -\frac{4\pi a \delta(t)}{f_{01}(x) - f_{02}(x)} \cos(\omega t).$$

4. APPLICATION TO PHYSICAL SYSTEMS

Now, we use the approach presented above to analyze the Duffing–Holmes-oscillator and pendulum equations. Transverse intersections of stable and unstable manifolds of these unperturbed systems give rise to homoclinic or heteroclinic orbits.

4.1. Duffing–Holmes Oscillator

The forced Duffing–Holmes oscillator with a parametrically perturbed cubic term is described by the equation

$$\ddot{x} - x + \beta[1 + \eta\cos(\Omega t)]x^3 = \varepsilon[\gamma\cos(\omega t) - \alpha\dot{x}], \quad (16)$$

where η and Ω are the amplitude and frequency of the parametric perturbation, respectively, and $\eta \ll 1$. We rewrite it as

$$\begin{aligned} \dot{x} &= v, \\ \dot{v} &= x - \beta x^3 + \varepsilon[\gamma\cos(\omega t) - \beta\eta x^3 \cos(\Omega t) - \alpha v]. \end{aligned} \quad (17)$$

The corresponding unperturbed Hamiltonian is

$$H_0 = \frac{v^2}{2} - \frac{x^2}{2} + \frac{\beta x^4}{4}.$$

Setting H_0, we find that system (17) has a single hyperbolic point ($x = v = 0$) with a single separatrix. The solution on the separatrix can be represented as [21] (see also [12–15])

$$x_0(t) = \frac{\sqrt{2}}{\sqrt{\beta}} \cosh t, \quad (18)$$

$$v_0(t) = \dot{x}_0(t) = -\frac{\sqrt{2}}{\sqrt{\beta}} \frac{\sinh t}{\cosh^2 t}. \quad (19)$$

Comparing this system with (1), we write

$$f_{01} = v, \quad f_{11} = 0,$$

$$f_{02} = x - \beta x^3,$$

$$f_{12} = \gamma\cos(\omega t) - \eta\beta x^3 \cos(\Omega t) - \alpha v.$$

Therefore,

$$f_0 \wedge f_1 = v_0[\gamma\cos(\omega t) - \eta\beta x_0^3 \cos(\Omega t) - \alpha v_0]$$

and (2) becomes

$$D(t, t_0) = -\int_{-\infty}^{+\infty} dt[\gamma v_0(t-t_0)\cos(\omega t)$$
$$-\eta\beta x_0^3(t-t_0)v_0(t-t_0)\cos(\Omega t) - \alpha v_0^2(t-t_0)]. \quad (20)$$

Changing to the integration variable $\tau = t - t_0$, we finally obtain [12–15]

$$D(t, t_0) = \frac{2\sqrt{2}}{\sqrt{\beta}}\pi\gamma\omega\frac{\sin(\omega t_0)}{\cosh(\pi\omega/2)}$$
$$-\frac{\pi\eta}{6\beta}(\Omega^4 + 4\Omega^2)\frac{\sin(\Omega t_0)}{\sinh(\pi\Omega/2)} + \frac{4\alpha}{3\beta}. \quad (21)$$

The sign of $D(t, t_0)$ is preserved if

$$\frac{6\beta d\cosh(\pi\Omega/2)}{\pi(\Omega^4 + 4\Omega^2)} = \eta_{min} < \eta \le \eta_{max}$$
$$= \frac{1}{p^2}\frac{6\sqrt{2\beta}\gamma\omega}{(\Omega^4 + 4\Omega^2)}\frac{\sinh(\pi\Omega/2)}{\cosh(\pi\omega/2)}, \quad (22)$$

where p is an integer (see [12–15]). Using the left-hand inequality in (22), we determine the critical value of the Melnikov function:

$$D_c(t, t_0) = \frac{2\sqrt{2}}{\sqrt{\beta}}\frac{\pi\gamma\omega}{\cosh(\pi\omega/2)}\sin(\omega t_0)$$
$$+\frac{4\alpha}{3\beta} - d\sin(\Omega t_0).$$

(An analogous calculation can be performed for the right-hand inequality.)

Then, a subcritical value

$$D_{out}(t, t_0) < D_c(t, t_0).$$

can be represented as

$$D_{out}(t, t_0) = \frac{2\sqrt{2}}{\sqrt{\beta}}\frac{\pi\gamma\omega}{\cosh(\pi\omega/2)}\sin(\omega t_0)$$
$$+\frac{4\alpha}{3\beta} - d\sin(\Omega t_0) - a,$$

where $a > 0$ is a constant.

Since the perturbation required to regularize the dynamics of system (16) has the form

$$f^*(\Omega, t) = \mathrm{Re}\{e^{i\Omega t}A(t)\},$$

the corresponding Melnikov distance

$$D^*(t, t_0) = -\int_{-\infty}^{+\infty} v_0(t) f^*(\Omega, t)dt.$$

is

$$D^*(t, t_0) = -\int_{-\infty}^{+\infty} A(t)v_0(t)e^{i\Omega t}dt. \quad (23)$$

To find $A(t)$, we define

$$D^*(t, t_0) + D_{out}(t, t_0) \equiv D'(t, t_0).$$

Since the perturbation $f^*(\Omega, t)$ is regularizing by assumption, it holds that

$$D'(t, t_0) > D_c(t, t_0).$$

It is obvious that we can take any $D'(t, t_0)$ a fortiori greater than $D_c(t, t_0)$:

$$D'(t, t_0) = \frac{2\sqrt{2}}{\sqrt{\beta}}\frac{\pi\gamma\omega}{\cosh(\pi\omega/2)}\sin(\omega t_0)$$
$$+\frac{4\alpha}{3\beta} - d\sin(\Omega t_0) + a. \quad (24)$$

On the other hand, we can use (23) to write

$$D'(t, t_0) = -\int_{-\infty}^{+\infty} A(t)v_0(t)e^{-i\Omega t}dt$$
$$+\frac{2\sqrt{2}}{\sqrt{\beta}}\frac{\pi\gamma\omega}{\cosh(\pi\omega/2)}\sin(\omega t_0) + \frac{4\alpha}{3\beta} - d\sin(\Omega t_0) - a. \quad (25)$$

Equating (24) to (25), we have

$$\int_{-\infty}^{+\infty} A(t)v_0(t)e^{i\Omega t}dt = -2a.$$

The inverse Fourier transform yields

$$A(t) = -\frac{2a}{v_0(t)}\int_{-\infty}^{+\infty} e^{-i\Omega t}d\Omega.$$

Therefore, dynamics of the forced Duffing–Holmes oscillator are regularized by the perturbation

$$f^*(\Omega, t) = -\frac{4\pi a\delta(t)}{v_0(t-t_0)}\cos(\Omega t). \quad (26)$$

4.2. Pendulum

The analysis presented above can be extended to the classical nonlinear pendulum, whose separatrices make up a heteroclinic orbit in the absence of damping. A periodically forced, damped pendulum is described by the equation [22]

$$\ddot{x} + \alpha\dot{x} + \sin x = \gamma\cos(\omega t). \quad (27)$$

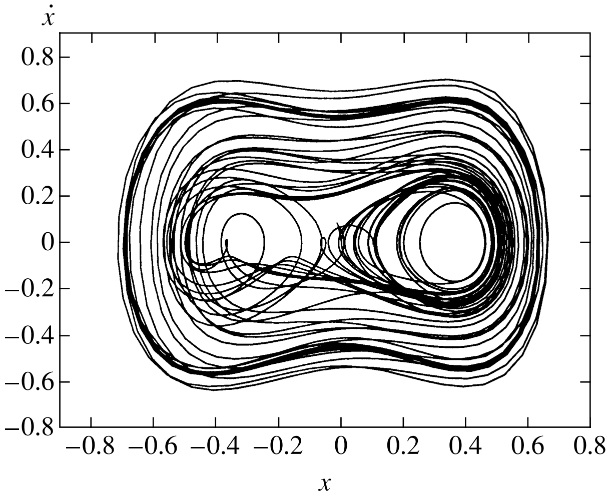

Fig. 2. Phase portrait of Duffing–Holmes oscillator (16): $\alpha = 0.145$, $\beta = 8$, $\eta = 0.03$, $\gamma = 0.14$, $\Omega = \omega = 1.1$.

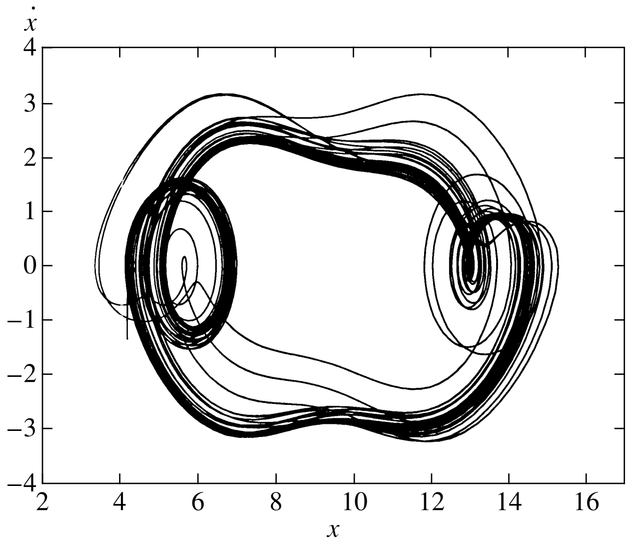

Fig. 3. Phase portrait of pendulum (27): $\alpha = 0.04$, $\gamma = 1.35$, $\omega = 1.0$.

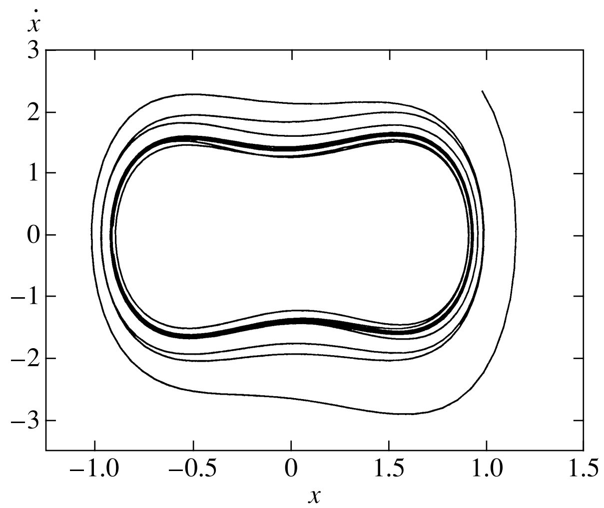

Fig. 4. Phase portrait of Duffing–Holmes oscillator (31): $\alpha = 0.145$, $\beta = 8$, $\eta = 0.03$, $\gamma = 0.14$, $\Omega = \omega = 1.1$, $a = 2$.

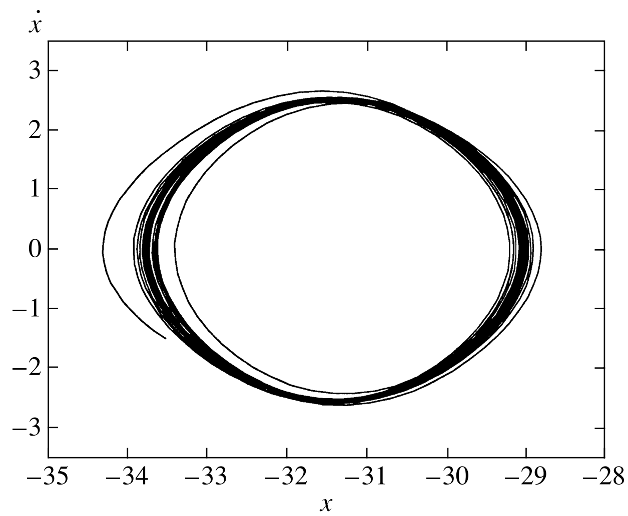

Fig. 5. Phase portrait of pendulum (32): $\alpha = 0.04$, $\gamma = 1.35$, $\omega = 1.0$, $a = 1.2$.

The corresponding unperturbed Hamiltonian is

$$H_0 = \frac{\dot{x}^2}{2} - \cos x.$$

The phase portrait of the pendulum is 2π-periodic in x, with hyperbolic points at $(\pm\pi, x)$ and a center at (0, 0). The system has oscillatory, rotatory, and separatrix solutions. We focus here on solutions of the last type:

$$x_0(t) = \pm\frac{\tanh t}{\cosh t},$$

$$\dot{x}_0(t) = \pm\frac{2}{\cosh t}.$$

The Melnikov distance corresponding to (27) is [22]

$$D(t_0, \omega) = -\alpha\int_{-\infty}^{+\infty}(\dot{x}_0(t))^2 dt \pm \gamma\cos(\omega t_0)\int_{-\infty}^{+\infty}\sin(x_0(t))\dot{x}_0(t)\cos\omega t dt. \quad (28)$$

Calculating the integrals, we obtain

$$D(t_0, \omega) = -4\alpha B\left(\frac{1}{2}, 1\right) \pm \frac{2\pi\gamma}{\cosh\left(\frac{\pi\omega}{2}\right)}\cos(\omega t_0), \quad (29)$$

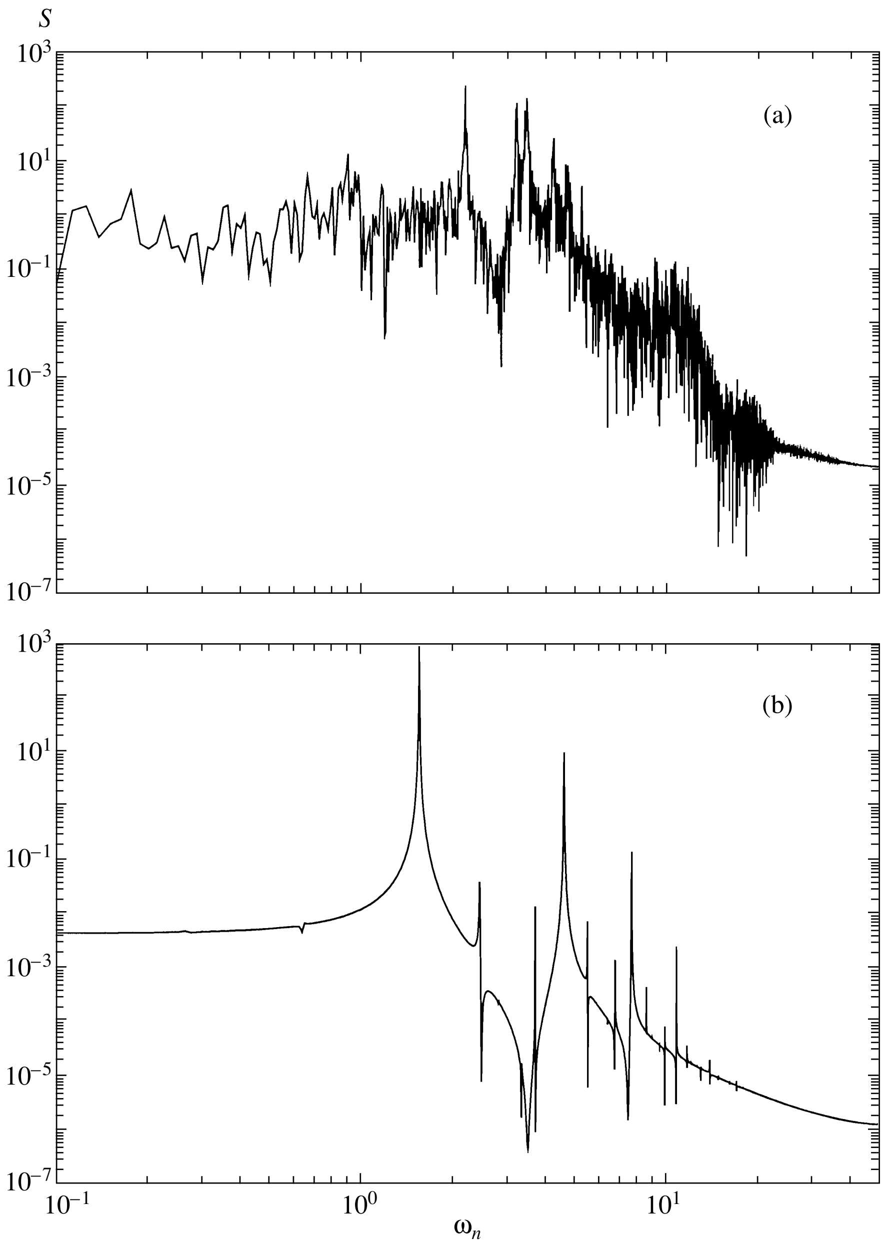

Fig. 6. Spectral density of a realization $x(t)$ for (a) original Duffing–Holmes oscillator (16) with $\alpha = 0.145$, $\beta = 8$, $\eta = 0.03$, $\gamma = 0.14$, and $\Omega = \omega = 1.1$ and (b) regularized Duffing–Holmes oscillator (31) with $a = 2$.

where $B(r, s)$ is Euler's beta function.

Since this Melnikov function $D(t_0, \omega)$ obviously admits additive shift from its critical values, chaotic behavior of the pendulum is suppressed by the perturbation

$$f^*(\omega, t) = -\frac{4\pi a\delta(t)}{\dot{x}_0(t-t_0)}\cos(\omega t), \qquad (30)$$

where $\dot{x}_0(t)$ is the solution on the unperturbed separatrix.

Physically, the results obtained here mean that dynamics of the Duffing–Holmes oscillator and pendulum are regularized by series of "kicks."

4.3. Numerical Results

In the preceding section, it is shown that chaos in Duffing–Holmes-oscillator and pendulum dynamics can be suppressed by applying perturbations (26) and (30), respectively. In this section, we present the results of a numerical analysis.

We consider Eqs. (16) and (27). In dynamics of the Duffing–Holmes oscillator, the onset of chaos corre-

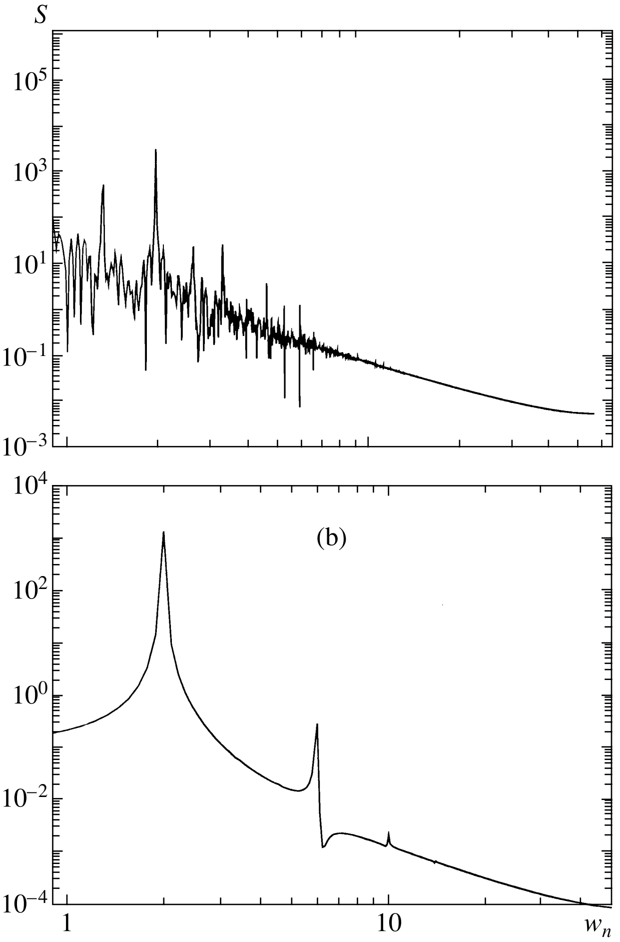

Fig. 7. Spectral density of a realization $x(t)$ for (a) original pendulum equation (27) with $\alpha = 0.04$, $\gamma = 1.35$, and $\omega = 1.0$ and (b) perturbed equation (32) with $a = 1.2$.

sponds to the breakdown of a figure-of-eight separatrix. Figure 2 illustrates the structure of a typical chaotic set obtained in this case. The onset of chaos in pendulum dynamics is associated with the breakdown of a heteroclinic trajectory (see Fig. 3).

Consider the Duffing–Holmes oscillator and pendulum with additional perturbations (26) and (30), respectively. The corresponding equations are

$$\ddot{x} - x + \beta[1 + \eta\cos(\Omega t)]x^3 = \varepsilon\Big[\gamma\cos(\omega t) - \alpha\dot{x} + 2\pi\sqrt{2\beta}\frac{\cosh^2(t-t_0)}{\sinh(t-t_0)}a\delta(t)\cos\Omega t\Big] \quad (31)$$

$$\ddot{x} + \alpha\dot{x} + \sin x = \gamma\cos(\omega t) + 2\pi\cosh(t-t_0)a\delta(t)\cos(\omega t). \quad (32)$$

Figures 4 and 5 show numerical solutions to systems (31) and (32), respectively. It is clear that the dynamics of both oscillator and pendulum approach regular regimes represented by periodic orbits.

To analyze systems (31) and (32) in more detail, we invoke the spectral density defined as

$$S(\omega) = \lim_{T\to\infty}\frac{1}{2\pi T}|X(\omega)|^2,$$

where $X(\omega)$ is the Fourier transform of a solution $x(t)$ to system (16) or (27). The spectral density provides a simple, but reliable characterization of dynamics of a system under study. It can readily be used to find out whether a motion is regular or chaotic.

Figures 6a and 7a show the spectral densities calculated for original systems (16) and (27), respectively; Figs. 6b and 7b, the spectral densities for systems subject to perturbations (26) and (30), respectively. These results demonstrate that chaos is suppressed and dynamics of both systems are regularized.

Taking different parameter values corresponding to chaotic behavior, one can find appropriate regularizing perturbations (see above) and obtain qualitatively similar results, i.e., change from chaotic states to regular oscillations.

Thus, our numerical analysis is consistent with the analytical results obtained in Section 3.

5. CONCLUSIONS

Separatrix splitting is a very convenient method for examining dynamical systems, because it can be used to obtain nonintegrability conditions for many applied problems in analytical form [23]. Currently, the problem of chaos suppression considered in this study is mainly solved by numerical methods (e.g., see [1–10]). However, asymptotic behavior of trajectories can be examined analytically. As a result, the distance between the separatrices split by a perturbation can be found in general form by applying a perturbation method in the vicinity of a homoclinic trajectory.

In this study, separatrix splitting is applied to explore the possibility of chaos suppression in dissipative systems. Analytical expressions are obtained for regularizing perturbations. These results are sufficiently general to be applied to various dynamical systems that admit separatrix splitting.

REFERENCES

1. S. Boccaletti, C. Grebogi, Y.-C. Lai, *et al.*, Phys. Rep. **329**, 103 (2000).
2. A. Loskutov and S. D. Rybalko, in *Proceedings of the 5th International Conference on Difference Equations and Applications, ICDEA'2000, Temuco, Chile, 2000* (Taylor and Francis, London, 2002), p. 207.
3. V. V. Alekseev and A. Yu. Loskutov, Vestn. Mosk. Univ., Ser. 3: Fiz. Astron. **26** (3), 40 (1985).

4. V. V. Alekseev and A. Yu. Loskutov, Dokl. Akad. Nauk SSSR **293**, 1346 (1987) [Sov. Phys. Dokl. **32**, 270 (1987)].
5. E. Ott, C. Grebogi, and J. A. Yorke, Phys. Rev. Lett. **64**, 1196 (1990).
6. F. J. Romeiras, E. Ott, C. Grebogi, and W. P. Dayawansa, Physica D (Amsterdam) **58**, 165 (1992).
7. T. Shinbrot, C. Grebogi, E. Ott, and J. A. Yorke, Nature **363**, 411 (1993).
8. T. Shinbrot, Adv. Phys. **44**, 73 (1995).
9. E. Ott and M. L. Spano, Phys. Today **48**, 34 (1995).
10. A. Loskutov, Comput. Math. Mod. **12**, 314 (2001).
11. V. K. Mel'nikov, Tr. Mosk. Mat. O–va **12**, 3 (1963).
12. R. Lima and M. Pettini, Phys. Rev. A **41**, 726 (1990).
13. F. Cuadros and R. Chacón, Phys. Rev. E **47**, 4628 (1993).
14. R. Lima and M. Pettini, Phys. Rev. E **47**, 4630 (1993).
15. R. Chacón, Phys. Rev. E *51*, 761 (1995).
16. Y. Kivshar, F. Rodelsperger, and H. Benner, Phys. Rev. E **49**, 319 (1994).
17. R. Chacón, Phys. Rev. Lett. **86**, 1737 (2001).
18. G. Cicogna and L. Fronzoni, Phys. Rev. A **42**, 1901 (1990).
19. V. G. Gel'freĭkh and V. F. Lazutkin, Usp. Mat. Nauk **56**, 79 (2001).
20. P. J. Holmes and J. E. Marsden, Commun. Math. Phys. **82**, 523 (1982).
21. J. Guckenheimer and P. J. Holmes, *Nonlinear Oscillations, Dynamical Systems and Bifurcation of Vector Fields* (Springer, Berlin, 1990).
22. J. L. Trueba, J. Rams, and M. A. F. Sanjuán, Int. J. Bifurcation Chaos Appl. Sci. Eng. **10**, 2257 (2000).
23. V. V. Kozlov, *Symmetry, Topology, and Resonances in the Hamilton Mechanics* (Udmurt. Gos. Univ., Izhevsk, 1995).

Translated by A. Betev

Spell: ok