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REPRODUCIBILITY OF THE STRUCTURE AND PROPERTIES OF PARTS AND THEIR DESCRIPTION WITHIN THE FRAMEWORK OF NONLINEAR DYNAMICS

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The problem of the reproducibility of complex structures is discussed from the standpoint of the applied theory of dynamic systems. Some fundamental concepts of the theory of nonlinear fluctuations (bifurcations, fractal sets, space and time networks of interacting subsystems) that can be helpful in applications are described on a qualitative level.

The problem of the reproducibility of complex structures that appear in nonlinear media has become closely connected with some problems of the applied theory of dynamic systems. Indeed, at the present time such concepts as bifurcation sets, fractals, strange attractors, analysis of time series, etc. penetrate the applied sciences deeper and deeper [1]. However, many aspects of the theory have not yet been comprehended in detail. Moreover, some of the concepts of the theory of dynamic systems are interpreted incorrectly in the applied aspect. It becomes more and more obvious that after the appearance of recent popular publications concerning achievements of nonlinear dynamics it has shared the fate of cybernetics and the theory of catastrophes: the deep works concern the respective special fields and the well-known terms have been preserved in speculative works only or, at best, in popular scientific literature. Such a situation is a consequence of misunderstanding of the basic concepts of the modern theory of dynamic systems. It is appropriate to recall here the words of Academician V. I. Arnol'd: "It is hard to agree with the fact that the introduction of a new term not accompanied by a discovery of new facts is a substantial achievement. However, the success of 'cybernetics,' 'attractors,' and the 'theory of catastrophes' reflects the fruitfulness of word-invention as a method of scientific work."

These words can be applied to the use of the results of nonlinear dynamics in particular cases. Let us discuss one such case, namely, the reproducibility of the quality of parts.

As a rule, reproducibility is understood as the capacity of a system to have the same "quality" in a steady state. Quality is commonly understood as a set of requirements imposed on the system. However, the situation is aggravated (sometimes quite substantially) if the system attains a steady state after some qualitatively different transition processes (bifurcation states). The main reason for reproducibility in this case is multistability rather than the presence of bifurcation points. Multistability [2] is understood as the presence of a system of coexisting attractors in the phase space. The properties of the system will depend on the attraction field of what attractor the initial conditions fall in.

An unsteady state is quite often understood as the position of the system at a bifurcation point. However, this is not correct in the general case. At a bifurcation point the system is unsteady indeed, but an unsteady system does not necessarily have to be in a bifurcation state. Many authors dealing with applied problems of nonlinear dynamics erroneously assume that the system at a bifurcation point is seemingly obliged to choose the subsequent evolution, but in the presence of noise (inevitable to this or that degree) this path is indeterminate in advance.

Let us consider a model example. Assume that we have some melt. If the temperature is maintained at a constant value, the properties of the melt will not change. However, with a specified decrease in the temperature (cooling) the melt will pass through a number of qualitative transformations, and the steady state of such a system (under normal conditions) will be a certain specimen. It is obvious that the bifurcation parameter in this example is the temperature. What determines the properties of the final state of the system (specimen)? It is natural that the observation of permissible deviations in the concentration of the components of the melt, the pressure, the rate of decrease of the temperature, etc. will be very important. Under identical conditions (even in the presence of low noise) the system will go through one and the same cascade of bifurcations upon a change in the parameter. At the same time, if the system (specimen) has several attractors in the final state, the concentration of the

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components of the melt will be the main technological criterion of reproducibility. Therefore, the reproducibility of the quality of the specimen in the given situation is connected foremost, if we use the language of nonlinear dynamics, with the multistability of the system.

Another situation can be encountered when the parameters of the system change slowly with time. In this case new phenomena appear, for example, disappearance of the equilibrium or its instability, so that the system has to pass rapidly to a new state, or extension of the loss of stability, etc. These phenomena are known as relaxation fluctuations (the mathematical side of the problem is described in [3]). Such fluctuations can be either regular or chaotic, which corresponds to random time intervals between subsequent transitions of the system from one regime of motion to another. Relaxation fluctuations in the problem of reproducibility play a role far from the last, and consideration of this problem with allowance for such fluctuations can give us a key to answering many questions.

On the other hand, the theory of bifurcations can be helpful for the solution of applied problems. This is connected with the fact that all systems of ordinary differential equations of the same dimensionality near the values of the parameters at which a bifurcation of one type occurs are topologically equivalent. Consequently, by describing a bifurcation and determining its type we can judge the behavior of the system in the neighborhood of the bifurcation value of the parameter. In addition to the widely known types of bifurcation, such as the bifurcation of Andronov – Hopf, the bifurcation of the origination of a torus, the bifurcation of doubling of a period, etc. we can quite often encounter bifurcations of contours composed of saddle separatrices [3 – 5]. The study of such bifurcations has been promoted by the discovery of homoclinic trajectories and separatrix contours in models important for applications. The well-known Lorenz model [2] having a homoclinic contour of the eight-butterfly type has played a rather significant role here. This system is the first example of discovery of chaotic motion.

Another direction that can be useful for solving problems of reproducibility in its applied sense is the theory of fractal sets [6, 7]. The theory of fractals had not been widely used for a long time until the discovery of problems where the fractal structure and the dimensionality are the main characteristics of the system. For example, the theory of fractals in turbulence is closely connected with Kolmogorov's theory of scale invariance. If we consider the speed of a turbulent flow as a function of the space variables and the time, it will we represented by a fractal of the same type as the Brownian curve.

Fractal sets occupy a special place in the theory of dynamic systems because the solutions of the majority of nonlinear problems have a fractal form. The point is that chaotic fluctuations in dissipative systems are representable mathematically by an attractor that does not possess such a smooth surface as, for example, a torus. The geometrical structure of such attractors is much more complex. In particular, they can possess a geometrical (scale) invariance, i.e., be representable by fractals. The complex geometry of strange attractors sometimes allows us to describe them just like fractals.

Fractal theory has common aspects with the method of the renormgroup and the theory of phase transformations. Important applications of the theory of fractal sets have been discovered unexpectedly in materials science, theoretical biology, mathematical modeling, and other fields. Many topics have been comprehended here based on scale invariance.

One of the methods for describing such complex objects as fractals is aggregation limited by diffusion [6]. In accordance with this mechanism a certain kind of fractal can be obtained in the process of random irreversible growth. The process is extremely nonequilibrium. However, it can be used successfully for explaining some properties of the growth of fractal structures.

In a real situation, a nucleus moves in accordance with some law. The development of this problem should give us an answer to questions connected with structure formation in nonequilibrium systems and create conditions for the formation of clusters with specified properties and specific dimensionality.

Structure formation in spatially extended chaotic media is one of the most interesting problems of nonlinear dynamics closely related to the problem of reproducibility. One of the methods of the description of such media consists in their approximation by a set of discrete elements interacting locally with each other. It is known that even when the individual elements of the medium possess a complex internal structure, their complexity does not manifest itself fully in the interactions between them, and they function with respect to the macrosystem as quite simple objects with a small number of effective degrees of freedom. As a rule, in the opposite case not a single ordered structure appears in the system. The problem of nonlinear dynamics consists in finding and investigating in detail fundamental mathematical models based on use of the most typical suppositions concerning the properties of the individual elements that constitute the system and the laws of interaction between them.

The studied medium can be quantized either with respect to the space or with respect to the space and the time. In the case of spatial quantization the initial system is approximated by a finite or countable set of elements with a certain coupling between them. Every such element is represented by a dynamic system with a small number of variables. If the dynamic system is additionally specified by a map, we speak of space-and-time quantization of the initial system. Space-and-time discrete models are known as lattice or network ones.

The space-and-time lattices of interacting systems can be one-dimensional, two-dimensional, or three-dimensional. The structures of the lattices can differ too. Map lattices are studied most often, when every element interacts in this or that way only with its closest neighbors. Another kind of coupling presumes global cohesion, when every element of the lattice is coupled with every neighbor. Local interaction predominantly studies the diffusion kind of coupling between the elements. If the whole of the lattice is composed of identical elements, it is called homogeneous. If the lattice contains "embeddings," it ceases to be homogeneous and is much harder to investigate. Homogeneous lattices are studied (commonly by numerical methods) in virtually any work devoted to diffusion-interacting maps [8 – 11]. However, it is obvious that in the applied aspect the homogeneity of a space (in our case the identical nature of all the elements) is an idealization used to simplify the analysis. Therefore, it is of interest to determine how the system will change qualitatively with the appearance of inhomogeneities.

The following questions arise in the study of inhomogeneous lattices. If the behavior of individual elements is chaotic, will the lattice manifest chaotic properties too? Will the chaos suppress the surrounding order or will the order of the individual elements propagate to the whole of the lattice? Moreover, why is a complex space-and-time state preferable for some nonlinear media to simple homogeneous behavior (i.e., when the system "spontaneously" transforms from a virtually homogeneous state to a spatially inhomogeneous one) and how can such a state be realized? These questions can be referred to problems of polymerization, creation of structures with specified properties, reproducibility, etc.

Intense research on chaotic dynamic systems has shown an unexpected and wonderful property: they are quite pliable and extremely sensitive to external actions [12 – 15]. It seems that this very fact is responsible for the processes of structure formation. The development of any system is a consequence of autonomous acts of self-organization. For this reason, the developing system can pass to one of a very large number of permissible states. However, the evolving system is always characterized by a specific (specified) dynamics. This process can be controlled with the help of weak actions that affect the choice of this or that state. Thus, it has been shown that it is possible to control the dynamics of complex systems, i.e., to transform initially chaotic systems from a regime of chaotic fluctuations to the requisite dynamic regime, and thus stabilize their behavior by means of quite weak actions.

Most systems are open; they are characterized by an energy exchange with the ambient. Therefore, we can assume that the lattice is subjected to an external action, i.e., that every element of it experiences a certain influence. In this case a specified space pattern will be preserved with a specific diffusion coefficient. However, qualitative restructuring occurs when the diffusion intensifies, i.e., the spatial order is distorted, and elements with both regular and chaotic dynamics begin to appear in the system in a random manner. With further intensification of the diffusion the behavior of the distributed system as a whole becomes chaotic [16]. Order is always absorbed by disorder.

Let us now assume that initially (no external action) we have a homogeneous distributed medium whose elements can manifest both regular and chaotic dynamics. If we assume that this system exchanges energy with the ambient, we can show that spatial clusters can appear in the system under the action of the arriving energy; the clusters consist of similarly functioning subsystems (maps). In other words, steady space-and-time structures appear in the open system. However, they can exist in a quite narrow range of diffusion coefficients; if their value is below the critical one, we are dealing with absolute order, if it exceeds the critical value, we are dealing with complete chaos.

Now let us assume that the elements do not possess rigid coupling but can arbitrarily collide with each other and wander chaotically over the accessible space. In this case the problem of structure formation can be reformulated in the language of cascades of nonlinear automata [17]. Studies of such cascades confirm the idea that random organization of a stable chain of elements is an extremely low-probability event. Only a small portion of the combinations of base elementary dynamic systems manifests a specified type of behavior. However, if we take into account that the reason behind the formation of a complex system is its capacity to manifest the prescribed dynamics, the number of possible combinations is reduced markedly. This fact allows us to suggest a mechanism lying at the basis of formation of complex structures that consist of a relatively small number of elementary subsystems.

Development of the described methods and knowledge of the rules of self-organization make it possible to create complexly organized distributed media with specified dynamic properties and to control their behavior. In turn, this can give a key to the solution of the problem of reproducibility of structures in complex space-and-time systems.

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