Control of a system with a strange attractor through periodic parametric action

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Certain facts indicating that rather simple systems demonstrate not only periodic dynamics but also very complex and irregular behavior have been known for more than twenty years.[1] However, it was only starting from the time the "strange attractor" concept introduced by Ruelle and Takens[2] was linked with the Lorenz model that there was hope that complicated effects in real systems, such as, for instance, turbulence, may be explained by using the "strange" attractor concept. Further studies of actual models showed that notwithstanding the extraordinary diversity of nonlinear physical systems the number of most widespread means of making the transition to irregular behavior when parameters are changed was quite small.[3,4] One encounters most often a transition through the bifurcation of period doubling. An important fact is they experimental discovery of these bifurcations in real systems. Hydrodynamics gives us beautiful examples of the formation of structures of ever increasing complexity: this is the observation of a hierarchy of instabilities between rotating cylinders[5] and in thermal convection.[6]

Can a system with a strange attractor be acted upon in a manner weak enough for it to behave predictably or to be controlled? Earlier studies[7,8] gave us some definite information.

In the present paper we show that by using a weak periodic action on the system parameters we can switch it from a stochasticity regime caused by the presence of a strange attractor into a periodic regime. We considered a set of equations describing the dynamics of a simple aqueous ecosystem consisting of two forms of microalgae and two forms of zooplanktons:

$$\frac{dM_1}{dt} = -\epsilon_1 M_1 - \gamma_1 \frac{M_1 M_2}{1 + aM_1} + \beta \frac{M_1 M_0}{1 + bM_0},$$
$$\frac{dM_2}{dt} = -\epsilon_2 M_2 + \gamma_2 \frac{M_1 M_2}{1 + aM_1},$$
$$\frac{dM_1'}{dt} = -\epsilon_1' M_1' - \gamma_1' \frac{M_1' M_2'}{1 + a' M_1'} + \beta' \frac{M_1' M_0}{1 + b' M_0},$$
$$\frac{dM_2'}{dt} = -\epsilon_2' M_2' + \gamma_2' \frac{M_1' M_2'}{1 + a' M_1'}. \qquad (1)$$

Here M_1, M_1' and M_2, M_2' are the biomasses of the forms of, respectively, the first (phytoplankton) and second (zooplankton) trophic level (the biomasses are expressed in units of the limiting biogenic element); $M_0 = M - (M_1 + M_2 + M_1' + M_2')$ is the amount of limiting biogenic element in the reservoir; M is the total mass of the limiting biogenic element in the whole system which is assumed to be constant; ϵ_1, ϵ_1', ϵ_2, and ϵ_2' are the mortality coefficients of the corresponding forms, γ_1, γ_1' and γ_2, γ_2' are, respectively, the coefficients of consumption and utilization by the forms of the second trophic level of forms of the first level; β and β' are photosynthesis coefficients; and a, a' and b, b' are saturation coefficients. In previous papers[9,10] we have shown that for well-defined parameters this system has a stochastic regime caused by the presence of a strange attractor in its phase space; this regime is obtained through successive period doublings.

Including the β in the free parameters, we found by means of a numerical experiment that when $\epsilon_1 = \epsilon_2 = \gamma_2 = \gamma_2' = 1$, $\gamma_1 = \epsilon_1' = \gamma_1' = \epsilon_2' = 2$, $a' = a = b = b' = 0.225$, and $M = 18$ in the region $\tilde{\beta}_1 = 0.99 < \beta = \beta' < 1.26 = \tilde{\beta}_2$ there is a self-stochastic regime. We show in Fig. 1 the projection of the

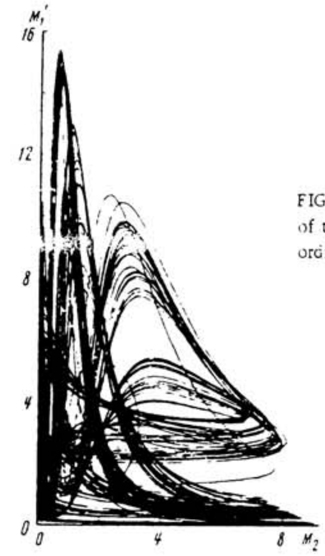

FIG. 1. Projection of the phase curve of the system (1) onto the (M_2, M_1') coordinate plane for $\beta_0 = 1.125$ and $\beta_1 = 0$.

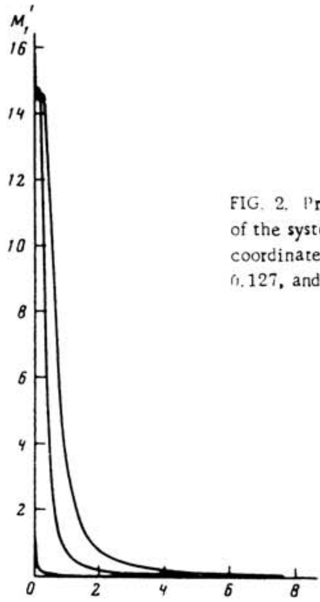

FIG. 2. Projection of the phase curve of the system (1) into the (M_2, M_1') coordinate plane for $\beta_0 = 1.125$, $\beta_1 = 0.127$, and $\omega = 1.277$.

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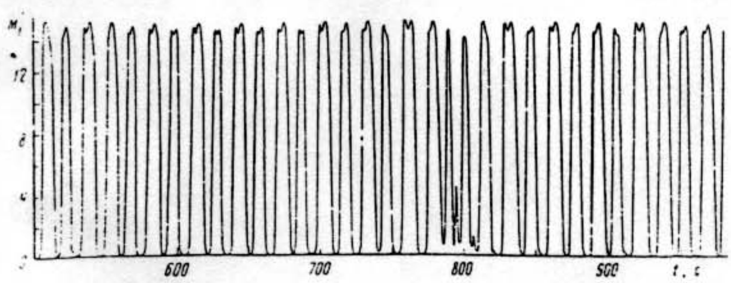

FIG. 3. The function $M_2(t)$ of the system (1) for $\beta_0 = 1.126$, $\beta_1 = -0.09$, and $\omega = \omega_-$.

phase curve which corresponds to that regime for $\beta = (\tilde{\beta}_1 + \tilde{\beta}_2)/2 = 1.125$. We assume that the parameter β in system (1) changes periodically with time, i.e., $\beta = \beta_0 + \beta_1 \sin \omega t$. In Ref. 8 it was shown that for $\beta_0 = 1.125$, $\beta_1 = 0.13$, and several frequencies ω of the system reaches a regular regime (destochastization). However, the β_1 dependence of the transition of the system to that regime was not studied there. From a physical point of view, it is clearly of most interest to find that range of variation of β_1 for which β does not leave the bounds of the stochasticity region, i.e., the region $|\beta_1| < (\tilde{\beta}_2 - \tilde{\beta}_1)/2$. Detailed analysis has shown that, indeed, for different values of β_1 lying in that range there is a parametric destochastization. The frequencies for which this is observed turn out to be close (within the second sign) to the natural frequencies of the linearized system (1) for the above-defined coefficients ϵ, γ, a, b, β, and M.

Figure 2 shows the phase portrait of system (1) for $\beta_0 = 1.125$, $\beta_1 = 0.127$, and $\omega = 1.277$. Similar results are also observed at the same frequency for smaller values of β_1. The threshold value of $|\beta_1|$, below which there is no transition to the regime of regular motion (for that value of ω), is $|\beta_1| \simeq 0.101$. One must at the same time note that for the threshold value of the parameter β_1 the locking into the regular regime in the parametric destochastization is slightly asymmetric relative to the initial phase of the perturbation. For the negative threshold value of β_1 the destochastization frequency is $\omega_- \simeq 1.274$, whereas for the positive threshold value of β_1 (for the same β_0) it is somewhat different. If we fix both β_0 and $\omega = \omega_\pm$, then by going beyond the threshold region $|\beta_1| < 0.101$, i.e., by decreasing the power of the parametric action, we see that starting from $|\beta_1| = 0.101$ the system will have a regime which is similar to a regime with intermittence (see Fig. 3). It is clear from the figure that at a certain time the system locks into a regular regime ($t \simeq 575$). At other, generally random times, there is a disruption, i.e., the system leaves the regular regime. A further lowering of β_1 brings on the stochastic regime in the system.

We wish to emphasize that a study of the behavior of system (1) as to stochasticity and regularity was in all cases carried out by us (numerically) using criteria such as the Kolmogorov entropy, Poincaré cycle mapping, and spectral analysis.

One can thus control a system which is in a stochasticity region by bringing it into a regular regime through a weak external periodic action.

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